CHAPTER 4
COMMON FRACTIONS

The emphasis in previous chapters of this course has been on integers (whole numbers). In this chapter, we turn our attention to numbers which are not integers. The simplest type of number other than an integer is a COMMON FRACTION. Common fractions and integers together comprise a set of numbers called the RATIONAL NUMBERS; this set is a subset of the set of real numbers.

The number line may be used to show the relationship between integers and fractions. For example, if the interval between 0 and 1 is marked off to form three equal spaces (thirds), then each space so formed is one-third of the total interval. If we move along the number line from 0 toward 1, we will have covered two of the three "thirds" when we reach the second mark. Thus the position of the second mark represents the number 2/3. (See fig. 4-1.)

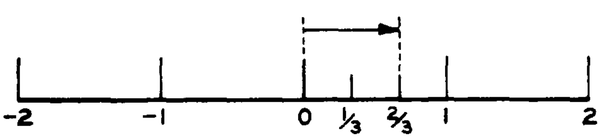


Figure 4-1.—Integers and fractions on the number line.

The numerals 2 and 3 in the fraction 2/3 are named so that we may distinguish between them; 2 is the NUMERATOR and 3 is the DENOMINATOR. In general, the numeral above the dividing line in a fraction is the numerator and the numeral below the line is the denominator. The numerator and denominator are the TERMS of the fraction. The word "numerator" is related to the word "enumerate." To enumerate means to "tell how many"; thus the numerator tells us how many fractional parts we have in the indicated fraction. To denominate means to "give a name" or "tell what kind"; thus the denominator tells us what kind of parts we have (halves, thirds, fourths, etc.).

Attempts to define the word "fraction" in mathematics usually result in a statement similar to the following: A fraction is an indicated division. Any division may be indicated by placing the dividend over the divisor and drawing a line between them. By this definition, any number which can be written as the ratio of two integers (one integer over the other) can be considered as a fraction. This leads to a further definition: Any number which can be expressed as the ratio of two integers is a RATIONAL number. Notice that every integer is a rational number, because we can write any integer as the numerator of a fraction having 1 as its denominator. For example, 5 is the same as 5/1. It should be obvious from the definition that every common fraction is also a rational number.

TYPES OF FRACTIONS

Fractions are often classified as proper or improper. A proper fraction is one in which the numerator is numerically smaller than the denominator. An improper fraction has a numerator which is larger than its denominator.

MIXED NUMBERS

When the denominator of an improper fraction is divided into its numerator, a remainder is produced along with the quotient, unless the numerator happens to be an exact multiple of the denominator. For example, 7/5 is equal to 1 plus a remainder of 2. This remainder may be shown as a dividend with 5 as its divisor, as follows:

$$\frac{7}{5} = \frac{5+2}{5} = 1 + \frac{2}{5}$$

The expression 1 + 2/5 is a MIXED NUMBER. Mixed numbers are usually written without showing the plus sign; that is, 1 + 2/5 is the same as $1\frac{2}{5}$ or 1 2/5. When a mixed number is written as 1 2/5, care must be taken to insure that there is a space between the 1 and the 2; otherwise, 1 2/5 might be taken to mean 12/5.

MEASUREMENT FRACTIONS

Measurement fractions occur in problems such as the following:

If $2 were spent for a stateroom rug at $3 per yard, how many yards were bought? If $6 had been spent we could find the number of yards by simply dividing the cost per yard into the amount spent. Since 6/3 is 2, two yards could be bought for $6. The same reasoning applies when $2 are spent, but in this case we can only indicate the amount purchased as the indicated division 2/3. Figure 4-2 shows a diagram for both the $6 purchase and the $2 purchase.

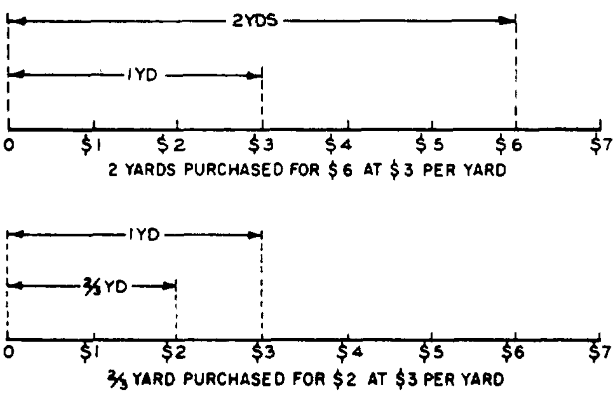


Figure 4-2.—Measurement fractions.

PARTITIVE FRACTIONS

The difference between measurement fractions and partitive fractions is explained as follows: Measurement fractions result when we determine how many pieces of a given size can be cut from a larger piece. Partitive fractions result when we cut a number of pieces of equal size from a larger piece and then determine the size of each smaller piece. For example, if 4 equal lengths of pipe are to be cut from a 3-foot pipe, what is the size of each piece? If the problem had read that 3 equal lengths were to be cut from a 6-foot pipe, we could find the size of each pipe by dividing the number of equal lengths into the overall length. Thus, since 6/3 is 2, each piece would be 2 feet long. By this same reasoning in the example, we divide the overall length by the number of equal parts to get the size of the individual pieces; that is, 3/4 foot. The partitioned 6-foot and 3-foot pipes are shown in figure 4-3.

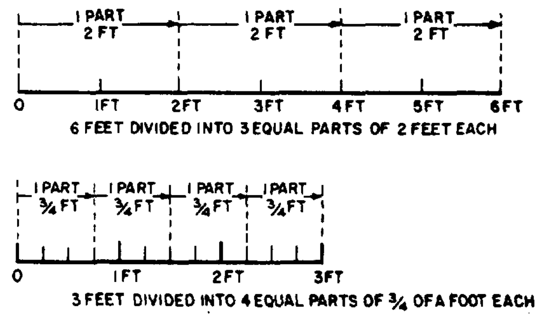


Figure 4-3.—Partitive fractions

EXPRESSING RELATIONSHIPS

When a fraction is used to express a relationship, the numerator and denominator take on individual significance. In this frame of reference, 3/4 means 3 out of 4, or 3 parts in 4, or the ratio of 3 to 4. For example, if 1 out of 3 of the men in a division are on liberty, then it would be correct to state that 1/3 of the division are on liberty. Observe that neither of these ways of expressing the relationship tells us the actual number of men; the relationship itself is the important thing.

Practice problems.

1. What fraction of 1 foot is 11 inches?
2. Represent 3 out of 8 as a fraction.
3. Write the fractions that indicate the relationship of 2 to 3; 8 divided by 9; and 6 out of 7 equal parts.
4. The number $6\frac{3}{5}$ means $6 \underline{\ ?\ } \frac{3}{5}$

Answers:

1. 11/12
2. 3/8
3. 2/3; 8/9; 6/7
4. plus

EQUIVALENT FRACTIONS

It will be recalled that any number divided by itself is 1. For example, 1/1, 2/2, 3/3, 4/4, and all other numbers formed in this way, have the value 1. Furthermore, any number multiplied by 1 is equivalent to the number itself. For example, 1 times 2 is 2, 1 times 3 is 3, 1 times 1/2 is 1/2, etc.

These facts are used in changing the form of a fraction to an equivalent form which is more convenient for use in a particular problem.

For example, if 1 in the form $\frac{2}{2}$ is multiplied by $\frac{3}{5}$, the product will still have a value of $\frac{3}{5}$ but will be in a different form, as follows:

$$\frac{2}{2} \cdot \frac{3}{5} = \frac{2 \cdot 3}{2 \cdot 5} = \frac{6}{10}$$

Figure 4-4 shows that $\frac{3}{5}$ of line a is equal to $\frac{6}{10}$ of line b where line a equals line b. Line a is marked off in fifths and line b is marked off in tenths. It can readily be seen that $\frac{6}{10}$ and $\frac{3}{5}$ measure distances of equal length.

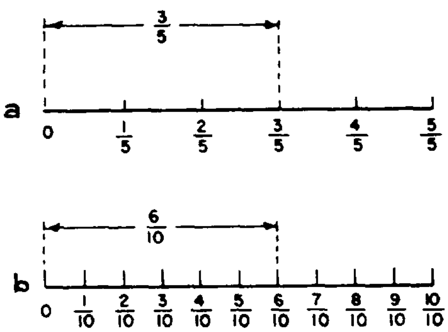


Figure 4-4.—Equivalent fractions.

The markings on a ruler show equivalent fractions. The major division of an inch divides it into two equal parts. One of these parts represents $\frac{1}{2}$. The next smaller markings divide the inch into four equal parts. It will be noted that two of these parts represent the same distance as $\frac{1}{2}$; that is, $\frac{2}{4}$ equals $\frac{1}{2}$. Also, the next smaller markings break the inch into 8 equal parts. How many of these parts are equivalent to $\frac{1}{2}$ inch? The answer is found by noting that $\frac{4}{8}$ equals $\frac{1}{2}$.

Practice problems. Using the divisions on a ruler for reference, complete the following exercise:

1. $\frac{1}{4} = \frac{?}{8}$
2. $\frac{1}{8} = \frac{?}{16}$
3. $\frac{3}{4} = \frac{?}{16}$
4. $\frac{1}{4} = \frac{?}{16}$

Answers:

1. 2
2. 2
3. 12
4. 4

A review of the foregoing exercise will reveal that in each case the right-hand fraction could be formed by multiplying both the numerator and the denominator of the left-hand fraction by the same number. In each case the number may be determined by dividing the denominator of the right-hand fraction by the denominator of the left-hand fraction. Thus in problem 1, both terms of $\frac{1}{4}$ were multiplied by 2. In problem 3, both terms were multiplied by 4. It is seen that multiplying both terms of a fraction by the same number does not change the value of the fraction.

Since $\frac{1}{2}$ equals $\frac{2}{4}$, the reverse must also be true; that is $\frac{2}{4}$ must be equal to $\frac{1}{2}$. This can likewise be verified on a ruler. We have already seen that $\frac{4}{8}$ is the same as $\frac{1}{2}$, $\frac{12}{16}$ equals $\frac{3}{4}$, and $\frac{2}{8}$ equals $\frac{1}{4}$. We see that dividing both terms of a fraction by the same number does not change the value of the fraction.

FUNDAMENTAL RULE OF FRACTIONS

The foregoing results are combined to form the fundamental rule of fractions, which is stated as follows: Multiplying or dividing both terms of a fraction by the same number does not change the value of the fraction. This is one of the most important rules used in dealing with fractions.

The following examples show how the fundamental rule is used:

1. Change 1/4 to twelfths. This problem is set up as follows:

$$\frac{1}{4} = \frac{?}{12}$$

The first step is to determine how many 4's are contained in 12. The answer is 3, so we know that the multiplier for both terms of the fraction is 3, as follows:

$$\frac{3}{3} \cdot \frac{1}{4} = \frac{3}{12}$$

2. What fraction with a numerator of 6 is equal to 3/4?

SOLUTION: $\frac{6}{?} = \frac{3}{4}$

We note that 6 contains 3 twice; therefore we need to double the numerator of the right-hand fraction to make it equivalent to the numerator of the fraction we seek. We multiply both terms of 3/4 by 2, obtaining 8 as the denominator of the new fraction, as follows:

$$\frac{6}{8} = \frac{3}{4} \cdot \frac{2}{2}$$

3. Change 6/16 to eighths.

SOLUTION: $\frac{6}{16} = \frac{?}{8}$

We note that the denominator of the fraction which we seek is 1/2 as large as the denominator of the original fraction. Therefore the new fraction may be formed by dividing both terms of the original fraction by 2, as follows:

$$\frac{6 \div 2}{16 \div 2} = \frac{3}{8}$$

Practice problems. Supply the missing number in each of the following:

1. $\frac{3}{8} = \frac{30}{?}$	3. $\frac{?}{90} = \frac{3}{10}$	5. $\frac{1}{?} = \frac{12}{72}$
2. $\frac{44}{48} = \frac{?}{12}$	4. $\frac{1}{6} = \frac{6}{?}$	6. $\frac{3}{5} = \frac{?}{25}$

Answers:

1. 80	3. 27	5. 6
2. 11	4. 36	6. 15

REDUCTION TO LOWEST TERMS

It is frequently desirable to change a fraction to an equivalent fraction with the smallest possible terms; that is, with the smallest possible numerator and denominator. This process is called REDUCTION. Thus, $\frac{6}{30}$ reduced to lowest terms is $\frac{1}{5}$. Reduction can be accomplished by finding the largest factor that is common to both the numerator and denominator and dividing both of these terms by it. Dividing both terms of the preceding example by 6 reduces the fraction to lowest terms. In computation, fractions should usually be reduced to lowest terms where possible.

If the greatest common factor cannot readily be found, any common factor may be removed and the process repeated until the fraction is in lowest terms: Thus, $\frac{18}{48}$ could first be divided by 2 and then by 3.

$$\frac{18 \div 2}{48 \div 2} = \frac{9}{24}$$

$$\frac{9 \div 3}{24 \div 3} = \frac{3}{8}$$

Practice problems. Reduce the following fractions to lowest terms:

1. $\frac{18}{48}$	2. $\frac{15}{20}$	3. $\frac{35}{56}$
4. $\frac{12}{60}$	5. $\frac{18}{24}$	6. $\frac{9}{144}$

Answers:

1. $\frac{3}{8}$	2. $\frac{3}{4}$	3. $\frac{5}{8}$
4. $\frac{1}{5}$	5. $\frac{3}{4}$	6. $\frac{1}{16}$

IMPROPER FRACTIONS

Although the "improper" fraction is really quite "proper" mathematically, it is usually customary to change it to a mixed number. A recipe may call for $1\frac{1}{2}$ cups of milk, but would not call for $\frac{3}{2}$ cups of milk.

Since a fraction is an indicated division, a method is already known for reduction of improper fractions to mixed numbers. The improper fraction $\frac{8}{3}$ may be considered as the division of 8 by 3. This division is carried out as follows:

$$\begin{array}{r l} & 2 \text{ R } 2 = 2\frac{2}{3} \\ 3 \big) & 8 \\ & \underline{6} \\ & 2 \end{array}$$

The truth of this can be verified another way: If 1 equals $\frac{3}{3}$, then 2 equals $\frac{6}{3}$. Thus,

$$2\frac{2}{3} = 2 + \frac{2}{3} = \frac{6}{3} + \frac{2}{3} = \frac{8}{3}$$

These examples lead to the following conclusion, which is stated as a rule: To change an improper fraction to a mixed number, divide the numerator by the denominator and write the fractional part of the quotient in lowest terms.

Practice problems. Change the following fractions to mixed numbers:

1. 31/20
2. 33/9
3. 65/20
4. 45/8

Answers:

1. $1\frac{11}{20}$
2. $3\frac{2}{3}$
3. $3\frac{1}{4}$
4. $5\frac{5}{8}$

OPERATING WITH MIXED NUMBERS

In computation, mixed numbers are often unwieldy. As it is possible to change any improper fraction to a mixed number, it is likewise possible to change any mixed number to an improper fraction. The problem can be reduced to the finding of an equivalent fraction and a simple addition.

EXAMPLE: Change $2\frac{1}{5}$ to an improper fraction.

SOLUTION:

Step 1: Write $2\frac{1}{5}$ as a whole number plus a fraction, $2 + \frac{1}{5}$.

Step 2: Change 2 to an equivalent fraction with a denominator of 5, as follows:

$$\frac{2}{1} = \frac{?}{5}$$

$$\frac{2(5)}{1(5)} = \frac{10}{5}$$

Step 3: Add $\frac{10}{5} + \frac{1}{5} = \frac{11}{5}$

Thus, $2\frac{1}{5} = \frac{11}{5}$

EXAMPLE: Write $5\frac{2}{9}$ as an improper fraction.

SOLUTION: $5\frac{2}{9} = 5 + \frac{2}{9}$

$$\frac{5}{1} = \frac{?}{9}$$

$$\frac{5(9)}{1(9)} = \frac{45}{9}$$

$$\frac{45}{9} + \frac{2}{9} = \frac{47}{9}$$

Thus, $5\frac{2}{9} = \frac{47}{9}$

In each of these examples, notice that the multiplier used in step 2 is the same number as the denominator of the fractional part of the original mixed number. This leads to the following conclusion, which is stated as a rule: To change a mixed number to an improper fraction, multiply the whole-number part by the denominator of the fractional part and add the numerator to this product. The result is the numerator of the improper fraction; its denominator is the same as the denominator of the fractional part of the original mixed number.

Practice problems. Change the following mixed numbers to improper fractions:

1. $1\frac{1}{5}$
2. $2\frac{11}{20}$
3. $3\frac{2}{7}$
4. $4\frac{3}{10}$

Answers:

1. $\frac{6}{5}$
2. $\frac{51}{20}$
3. $\frac{23}{7}$
4. $\frac{43}{10}$

NEGATIVE FRACTIONS

A fraction preceded by a minus sign is negative. Any negative fraction is equivalent to a positive fraction multiplied by -1. For example,

$$-\frac{2}{5} = -1\left(\frac{2}{5}\right)$$

The number $-\frac{2}{5}$ is read "minus two-fifths."

We know that the quotient of two numbers with unlike signs is negative. Therefore,

$$\frac{-2}{5} = -\frac{2}{5} \text{ and } \frac{2}{-5} = -\frac{2}{5}$$

This indicates that a negative fraction is equivalent to a fraction with either a negative numerator or a negative denominator.

The fraction $\frac{2}{-5}$ is read "two over minus five." The fraction $\frac{-2}{5}$ is read "minus two over five."

A minus sign in a fraction can be moved about at will. It can be placed before the numerator, before the denominator, or before the fraction itself. Thus,

$$\frac{-2}{5} = \frac{2}{-5} = -\frac{2}{5}$$

Moving the minus sign from numerator to denominator, or vice versa, is equivalent to multiplying the terms of the fraction by -1. This is shown in the following examples:

$$\frac{-2(-1)}{5(-1)} = \frac{2}{-5} \text{ and } \frac{2(-1)}{-5(-1)} = \frac{-2}{5}$$

A fraction may be regarded as having three signs associated with it—the sign of the numerator, the sign of the denominator, and the sign preceding the fraction. Any two of these signs may be changed without changing the value of the fraction. Thus,

$$-\frac{3}{4} = \frac{-3}{4} = \frac{3}{-4} = -\frac{-3}{-4}$$

OPERATIONS WITH FRACTIONS

It will be recalled from the discussion of denominate numbers that numbers must be of the same denomination to be added. We can add pounds to pounds, pints to pints, but not ounces to pints. If we think of fractions loosely as denominate numbers, it will be seen that the rule of likeness applies also to fractions. We can add eighths to eighths, fourths to fourths, but not eighths to fourths. To add $\frac{1}{5}$ inch to $\frac{2}{5}$ inch we simply add the numerators and retain the denominator unchanged. The denomination is fifths; as with denominate numbers, we add 1 fifth to 2 fifths to get 3 fifths, or $\frac{3}{5}$.

LIKE AND UNLIKE FRACTIONS

We have shown that like fractions are added by simply adding the numerators and keeping the denominator. Thus,

$$\frac{3}{8} + \frac{2}{8} = \frac{3+2}{8} = \frac{5}{8}$$

or

$$\frac{5}{16} + \frac{2}{16} = \frac{7}{16}$$

Similarly we can subtract like fractions by subtracting the numerators.

$$\frac{7}{8} - \frac{2}{8} = \frac{7-2}{8} = \frac{5}{8}$$

The following examples will show that like fractions may be divided by dividing the numerator of the dividend by the numerator of the divisor.

$$\frac{3}{8} \div \frac{1}{8} = ?$$

SOLUTION: We may state the problem as a question: "How many times does $\frac{1}{8}$ appear in $\frac{3}{8}$, or how many times may $\frac{1}{8}$ be taken from $\frac{3}{8}$?"

$$3/8 - 1/8 = 2/8 \qquad (1)$$

$$2/8 - 1/8 = 1/8 \qquad (2)$$

$$1/8 - 1/8 = 0/8 = 0 \qquad (3)$$

We see that 1/8 can be subtracted from 3/8 three times. Therefore,

$$3/8 \div 1/8 = 3$$

When the denominators of fractions are unequal, the fractions are said to be unlike. Addition, subtraction, or division cannot be performed directly on unlike fractions. The proper application of the fundamental rule, however, can change their form so that they become like fractions; then all the rules for like fractions apply.

LOWEST COMMON DENOMINATOR

To change unlike fractions to like fractions, it is necessary to find a COMMON DENOMINATOR and it is usually advantageous to find the LOWEST COMMON DENOMINATOR (LCD). This is nothing more than the least common multiple of the denominators.

Least Common Multiple

If a number is a multiple of two or more different numbers, it is called a COMMON MULTIPLE. Thus, 24 is a common multiple of 6 and 2. There are many common multiples of these numbers. The numbers 36, 48, and 54, to name a few, are also common multiples of 6 and 2.

The smallest of the common multiples of a set of numbers is called the LEAST COMMON MULTIPLE. It is abbreviated LCM. The least common multiple of 6 and 2 is 6. To find the least common multiple of a set of numbers, first separate each of the numbers into prime factors.

Suppose that we wish to find the LCM of 14, 24, and 30. Separating these numbers into prime factors we have

$$\begin{aligned} 14 &= 2 \cdot 7 \\ 24 &= 2^3 \cdot 3 \\ 30 &= 2 \cdot 3 \cdot 5 \end{aligned}$$

The LCM will contain each of the various prime factors shown. Each prime factor is used the greatest number of times that it occurs in any one of the numbers. Notice that 3, 5, and 7 each occur only once in any one number. On the other hand, 2 occurs three times in one number. We get the following result:

$$\begin{aligned} LCM &= 2^3 \cdot 3 \cdot 5 \cdot 7 \\ &= 840 \end{aligned}$$

Thus, 840 is the least common multiple of 14, 24, and 30.

Greatest Common Divisor

The largest number that can be divided into each of two or more given numbers without a remainder is called the GREATEST COMMON DIVISOR of the given numbers. It is abbreviated GCD. It is also sometimes called the HIGHEST COMMON FACTOR.

In finding the GCD of a set of numbers, separate the numbers into prime factors just as for LCM. The GCD is the product of only those factors that appear in all of the numbers. Notice in the example of the previous section that 2 is the greatest common divisor of 14, 24, and 30.

Find the GCD of 650, 900, and 700. The procedure is as follows:

$$\begin{aligned} 650 &= 2 \cdot 5^2 \cdot 13 \\ 900 &= 2^2 \cdot 3^2 \cdot 5^2 \\ 700 &= 2^2 \cdot 5^2 \cdot 7 \\ GCD &= 2 \cdot 5^2 = 50 \end{aligned}$$

Notice that 2 and 5^2 are factors of each number. The greatest common divisor is 2 x 25 = 50.

USING THE LCD

Consider the example

$$\frac{1}{2} + \frac{1}{3}$$

The numbers 2 and 3 are both prime; so the LCD is 6.

Therefore $\quad \frac{1}{2} = \frac{3}{6}$

and $\quad \frac{1}{3} = \frac{2}{6}$

Thus, the addition of $\frac{1}{2}$ and $\frac{1}{3}$ is performed as follows:

$$\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

In the example

$$\frac{1}{5} + \frac{3}{10}$$

10 is the LCD.

Therefore,
$$\frac{1}{5} + \frac{3}{10} = \frac{2}{10} + \frac{3}{10}$$
$$= \frac{5}{10} = \frac{1}{2}$$

Practice problems. Change the fractions in each of the following groups to like fractions with least common denominators:

1. $\frac{1}{3}, \frac{1}{6}$
2. $\frac{5}{12}, \frac{2}{3}$
3. $\frac{1}{2}, \frac{1}{4}, \frac{2}{3}$
4. $\frac{1}{6}, \frac{3}{10}, \frac{1}{5}$

Answers:

1. $\frac{2}{6}, \frac{1}{6}$
2. $\frac{5}{12}, \frac{8}{12}$
3. $\frac{6}{12}, \frac{3}{12}, \frac{8}{12}$
4. $\frac{5}{30}, \frac{9}{30}, \frac{6}{30}$

ADDITION

It has been shown that in adding like fractions we add the numerators. In adding unlike fractions, the fractions must first be changed so that they have common denominators. We apply these same rules in adding mixed numbers. It will be remembered that a mixed number is an indicated sum. Thus, $2\frac{1}{3}$ is really $2 + \frac{1}{3}$. Adding can be done in any order. The following examples will show the application of these rules:

EXAMPLE:

$$\begin{array}{r} 2\frac{1}{3} \\ 3\frac{1}{3} \\ \hline 5\frac{2}{3} \end{array}$$

This could have been written as follows:

$$\begin{array}{l} 2 + \frac{1}{3} \\ 3 + \frac{1}{3} \\ \hline 5 + \frac{2}{3} = 5\frac{2}{3} \end{array}$$

EXAMPLE:

$$\begin{array}{r} 4\frac{5}{7} \\ 6\frac{3}{7} \\ \hline 10\frac{8}{7} \end{array}$$

Here we change $\frac{8}{7}$ to the mixed number $1\frac{1}{7}$. Then

$$10\frac{8}{7} = 10 + 1 + \frac{1}{7}$$
$$= 11\frac{1}{7}$$

EXAMPLE:

Add
$$\begin{array}{r} \frac{1}{4} \\ 2\frac{2}{3} \\ \hline \end{array}$$

We first change the fractions so that they are like and have the least common denominator and then proceed as before.

$$\begin{array}{rcr} \frac{1}{4} & = & \frac{3}{12} \\ 2\frac{2}{3} & = & 2\frac{8}{12} \\ \hline & & 2\frac{11}{12} \end{array}$$

EXAMPLE:

Add
$$\begin{array}{rcr} 4\frac{5}{8} & = & 4\frac{5}{8} \\ 2\frac{1}{2} & = & 2\frac{4}{8} \\ \frac{1}{4} & = & \frac{2}{8} \\ \hline & & 6\frac{11}{8} \end{array}$$

Since $\frac{11}{8}$ equals $1\frac{3}{8}$, the final answer is found as follows:

$$6\frac{11}{8} = 6 + 1 + \frac{3}{8}$$
$$= 7\frac{3}{8}$$

Practice problems. Add, and reduce the sums to simplest terms:

1. $1\frac{1}{7}$	2. $\frac{3}{4}$	3. $6\frac{2}{5}$	4. $\frac{5}{8}$	5. $4\frac{1}{2}$
$2\frac{3}{4}$	$1\frac{1}{2}$	$3\frac{1}{4}$	$2\frac{3}{20}$	$1\frac{1}{8}$

Answers:

1. $3\frac{25}{28}$ 2. $2\frac{1}{4}$ 3. $9\frac{13}{20}$ 4. $2\frac{31}{40}$ 5. $5\frac{5}{8}$

The following example demonstrates a practical application of addition of fractions:

EXAMPLE: Find the total length of the piece of metal shown in figure 4-5 (A).

SOLUTION: First indicate the sum as follows:

$$\frac{9}{16} + \frac{3}{4} + \frac{7}{8} + \frac{3}{4} + \frac{9}{16} = ?$$

Changing to like fractions and adding numerators,

$$\frac{9}{16} + \frac{12}{16} + \frac{14}{16} + \frac{12}{16} + \frac{9}{16} = \frac{56}{16}$$
$$= 3\frac{8}{16}$$
$$= 3\frac{1}{2}$$

The total length is $3\frac{1}{2}$ inches.

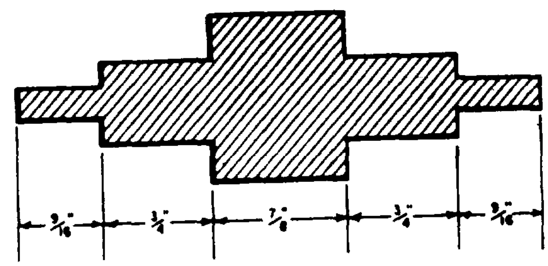
Practice problem. Find the distance from the center of the first hole to the center of the last hole in the metal plate shown in figure 4-5 (B).

Answer: $2\frac{7}{16}$ inches

SUBTRACTION

The rule of likeness applies in the subtraction of fractions as well as in addition. Some examples will show that cases likely to arise may be solved by use of ideas previously developed.

(A)



(B)

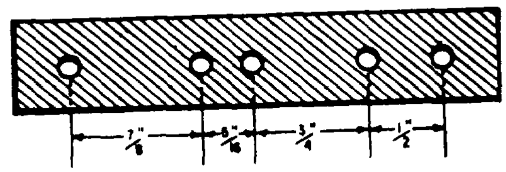


Figure 4-5.—Adding fractions to obtain total length or spacing.

EXAMPLE: Subtract $1\frac{1}{3}$ from $5\frac{2}{3}$

$$\begin{array}{r} 5\frac{2}{3} \\ 1\frac{1}{3} \\ \hline 4\frac{1}{3} \end{array}$$

We see that whole numbers are subtracted from whole numbers; fractions from fractions.

EXAMPLE: Subtract $\frac{1}{8}$ from $\frac{4}{5}$

$$\begin{array}{c} \frac{4}{5} \\ \frac{1}{8} \\ \hline \end{array}$$

Changing to like fractions with an LCD, we have

$$\begin{array}{c} \frac{32}{40} \\ \frac{5}{40} \\ \hline \frac{27}{40} \end{array}$$

EXAMPLE: Subtract $\frac{11}{12}$ from $3\frac{2}{3}$

$$3\frac{2}{3} = 3\frac{8}{12}$$

$$\frac{11}{12} = \frac{11}{12}$$

Regrouping $3\frac{8}{12}$ we have

$$2 + 1 + \frac{8}{12} = 2 + \frac{12}{12} + \frac{8}{12}$$

Then

$$3\frac{2}{3} = 2\frac{20}{12}$$

$$\frac{11}{12} = \frac{11}{12}$$

$$2\frac{9}{12} = 2\frac{3}{4}$$

Practice problems. Subtract the lower number from the upper number and reduce the difference to simplest terms:

1. $\frac{7}{9}$	2. $\frac{2}{3}$	3. $5\frac{5}{12}$	4. 5	5. $2\frac{3}{8}$
$\frac{1}{6}$	$\frac{1}{3}$	$2\frac{7}{12}$	$2\frac{2}{3}$	$\frac{5}{8}$

Answers:

1. $\frac{11}{18}$ 2. $\frac{1}{3}$ 3. $2\frac{5}{6}$ 4. $2\frac{1}{3}$ 5. $1\frac{3}{4}$

The following problem demonstrates subtraction of fractions in a practical situation.

EXAMPLE: What is the length of the dimension marked X on the machine bolt shown in figure 4-6 (A)?

SOLUTION: Total the lengths of the known parts.

$$\frac{1}{4} + \frac{1}{64} + \frac{1}{2} = \frac{16}{64} + \frac{1}{64} + \frac{32}{64} = \frac{49}{64}$$

Subtract this sum from the overall length.

$$2 = 1\frac{64}{64}$$

$$\frac{49}{64} = \frac{49}{64}$$

$$1\frac{15}{64}$$

The answer is $1\frac{15}{64}$ inch.

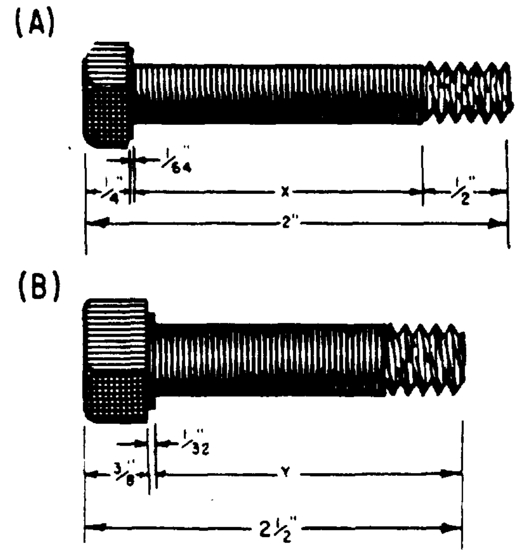


Figure 4-6.—Finding unknown dimensions by subtracting fractions.

Practice problem. Find the length of the dimension marked Y on the machine bolt in figure 4-6 (B).

Answer: $2\frac{3}{32}$ inches

MULTIPLICATION

The fact that multiplication by a fraction does not increase the value of the product may confuse those who remember the definition of multiplication presented earlier for whole numbers. It was stated that 4(5) means 5 is taken as an addend 4 times. How is it then that $\frac{1}{2}(4)$ is 2, a number less than 4? Obviously our idea of multiplication must be broadened.

Consider the following products:

$$4(4) = 16$$
$$3(4) = 12$$
$$2(4) = 8$$
$$1(4) = 4$$

$$\frac{1}{2}(4) = 2$$

$$\frac{1}{4}(4) = 1$$

Notice that as the multiplier decreases, the product decreases, until, when the multiplier is a fraction, the product is less than 4 and continues to decrease as the fraction decreases. The fraction introduces the "part of" idea: $\frac{1}{2}(4)$ means $\frac{1}{2}$ of 4; $\frac{1}{4}(4)$ means $\frac{1}{4}$ of 4.

The definition of multiplication stated for whole numbers may be extended to include fractions. Since 4(5) means that 5 is to be used 4 times as an addend, we can say that with fractions the numerator of the multiplier tells how many times the numerator of the multiplicand is to be used as an addend. By the same reasoning, the denominator of the multiplier tells how many times the denominator of the multiplicand is to be used as an addend. The following examples illustrate the use of this idea:

1. The fraction $\frac{1}{12}$ is multiplied by the whole number 4 as follows:

$$4 \times \frac{1}{12} = \frac{4}{1} \times \frac{1}{12}$$
$$= \frac{1 + 1 + 1 + 1}{12}$$
$$= \frac{4}{12} = \frac{1}{3}$$

This example shows that 4 (1/12) is the same as $\frac{4(1)}{12}$.

Another way of thinking about the multiplication of 1/12 by 4 is as follows:

$$4 \times \frac{1}{12} = \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12}$$
$$= \frac{4}{12} = \frac{1}{3}$$

2. The fraction 2/3 is multiplied by 1/2 as follows:

$$\frac{1}{2} \times \frac{2}{3} = \frac{2}{6}$$
$$= \frac{1}{3}$$

From these examples a general rule is developed: To find the product of two or more fractions multiply their numerators together and write the result as the numerator of the product; multiply their denominators and write the result as the denominator of the product; reduce the answer to lowest terms.

In using this rule with whole numbers, write each whole number as a fraction with 1 as the denominator. For example, multiply 4 times 1/12 as follows:

$$4 \times \frac{1}{12} = \frac{4}{1} \times \frac{1}{12}$$
$$= \frac{4}{12} = \frac{1}{3}$$

In using this rule with mixed numbers, rewrite all mixed numbers as improper fractions before applying the rule, as follows:

$$2\frac{1}{3} \times \frac{1}{2} = \frac{7}{3} \times \frac{1}{2}$$
$$= \frac{7}{6}$$

A second method of multiplying mixed numbers makes use of the distributive law. This law states that a multiplier applied to a two-part expression is distributed over both parts. For example, to multiply $6\frac{1}{3}$ by 4 we may rewrite $6\frac{1}{3}$ as 6 + 1/3. Then the problem can be written as 4(6 + 1/3) and the multiplication proceeds as follows:

$$4(6 + 1/3) = 24 + 4/3$$
$$= 25 + 1/3$$
$$= 25\frac{1}{3}$$

Cancellation

Computation can be considerably reduced by dividing out (CANCELLING) factors common to both the numerator and the denominator. We recognize a fraction as an indicated division. Thinking of $\frac{6}{9}$ as an indicated division, we remember that we can simplify division by showing both dividend and divisor as the indicated products of their factors and then dividing like factors, or canceling. Thus,

$$\frac{6}{9} = \frac{2 \times 3}{3 \times 3}$$

Dividing the factor 3 in the numerator by 3 in the denominator gives the following simplified result:

$$\frac{2 \times \overset{1}{\cancel{3}}}{3 \times \underset{1}{\cancel{3}}} = \frac{2}{3}$$

This method is most advantageous when done before any other computation. Consider the example,

$$\frac{1}{3} \times \frac{3}{2} \times \frac{2}{5}$$

The product in factored form is

$$\frac{1 \times 3 \times 2}{3 \times 2 \times 5}$$

Rather than doing the multiplying and then reducing the result $\frac{6}{30}$, it is simpler to cancel like factors first, as follows:

$$\frac{1 \times \overset{1}{\cancel{3}} \times \overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}} \times \underset{1}{\cancel{2}} \times 5} = \frac{1}{5}$$

Likewise,

$$\frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{\overset{1}{\cancel{2}}}{\cancel{6}}}{\underset{\underset{1}{\cancel{2}}}{\cancel{4}}} \times \frac{5}{9} = \frac{5}{9}$$

Here we mentally factor 6 to the form 3 x 2, and 4 to the form 2 x 2. Cancellation is a valuable tool in shortening operations with fractions.

The general rule may be applied to mixed numbers by simply changing them to improper fractions.

Thus,

$$2\frac{1}{4} \times 3\frac{1}{3} = ?$$

$$\frac{9}{4} \times \frac{10}{3} = \frac{\overset{3}{\cancel{9}}}{\underset{2}{\cancel{4}}} \times \frac{\overset{5}{\cancel{10}}}{\underset{1}{\cancel{3}}} = \frac{15}{2}$$

$$= 7\frac{1}{2}$$

Practice problems. Determine the following products, using the general rule and canceling where possible:

1. $\frac{5}{8} \times 12$	3. $5 \times \frac{4}{9}$	5. $\frac{1}{3} \times \frac{2}{3}$
2. $\frac{1}{2} \times \frac{1}{3} \times \frac{2}{5}$	4. $\frac{3}{4} \times 6$	6. $\frac{4}{3} \times \frac{1}{6}$

Answers:

1. $7\frac{1}{2}$	3. $2\frac{2}{9}$	5. $\frac{2}{9}$
2. $\frac{1}{15}$	4. $4\frac{1}{2}$	6. $\frac{2}{9}$

The following problem illustrates the multiplication of fractions in a practical situation.

EXAMPLE: Find the distance between the center lines of the first and fifth rivets connecting the two metal plates shown in figure 4-7 (A).

SOLUTION: The distance between two adjacent rivets, centerline to centerline, is 4 1/2 times the diameter of one of them.

Thus,

$$1 \text{ space} = 4\frac{1}{2} \times \frac{5}{8}$$

$$= \frac{9}{2} \times \frac{5}{8}$$

$$= \frac{45}{16}$$

There are 4 such spaces between the first and fifth rivets. Therefore, the total distance, D, is found as follows:

$$D = \overset{1}{\cancel{4}} \times \frac{45}{\underset{4}{\cancel{16}}} = \frac{45}{4} = 11\frac{1}{4}$$

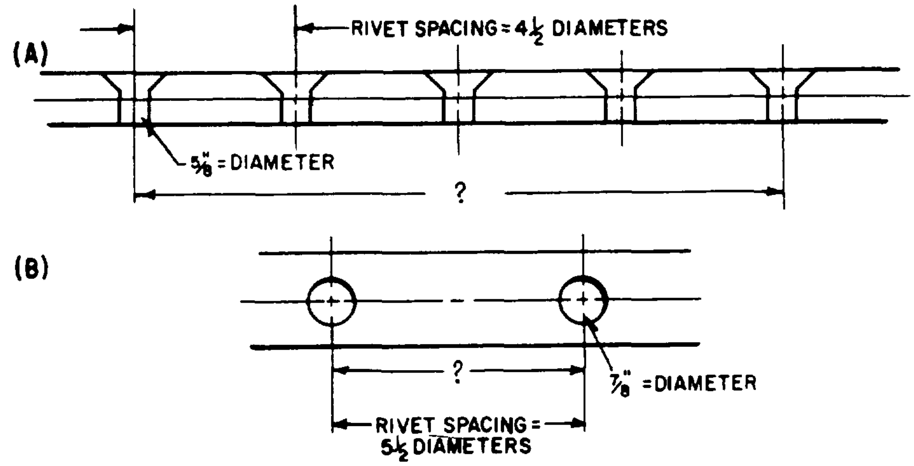


Figure 4-7.—Application of multiplication of fractions in determining rivet spacing.

The distance is $11\frac{1}{4}$ inches

Practice problem. Find the distance between the centers of the two rivets shown in figure 4-7 (B).

Answer: $4\frac{13}{16}$ inches

DIVISION

There are two methods commonly used for performing division with fractions. One is the common denominator method and the other is the reciprocal method.

Common Denominator Method

The common denominator method is an adaptation of the method of like fractions. The rule is as follows: Change the dividend and divisor to like fractions and divide the numerator of the dividend by the numerator of the divisor. This method can be demonstrated with whole numbers, first changing them to fractions with 1 as the denominator. For example, $12 \div 4$ can be written as follows:

$$12 \div 4 = \frac{12}{1} \div \frac{4}{1}$$

$$= \frac{12 \div 4}{1 \div 1}$$

$$= \frac{12 \div 4}{1}$$

$$= 3$$

If the dividend and divisor are both fractions, as in 1/3 divided by 1/4, we proceed as follows:

$$\frac{1}{3} \div \frac{1}{4} = \frac{4}{12} \div \frac{3}{12}$$

$$= \frac{4 \div 3}{12 \div 12}$$

$$= \frac{4 \div 3}{1}$$

$$= 4 \div 3 = 1\frac{1}{3}$$

Reciprocal Method

The word "reciprocal" denotes an interchangeable relationship. It is used in mathematics to describe a specific relationship between two numbers. We say that two numbers are reciprocals of each other if their product is one. In the example $4 \times \frac{1}{4} = 1$, the fractions $\frac{4}{1}$ and $\frac{1}{4}$ are reciprocals. Notice the interchangeability: 4 is the reciprocal of $\frac{1}{4}$ and $\frac{1}{4}$ is the reciprocal of 4.

What is the reciprocal of $\frac{3}{7}$? It must be a number which, when multiplied by $\frac{3}{7}$, produces the product, 1. Therefore,

$$\frac{3}{7} \times ? = 1$$

$$\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{7}}} \times \frac{\overset{1}{\cancel{7}}}{\underset{1}{\cancel{3}}} = 1$$

We see that $\frac{7}{3}$ is the only number that could fulfill the requirement. Notice that the numerator and denominator of $\frac{3}{7}$ were simply interchanged to get its reciprocal. If we know a number, we can always find its reciprocal by dividing 1 by the number. Notice this principle in the following examples:

1. What is the reciprocal of 7?

$$1 \div 7 = \frac{1}{7}$$

Check:

$$\frac{\cancel{7}}{\cancel{1}} \times \frac{\cancel{1}}{\cancel{7}} = 1$$

Notice that the cancellation process in this example does not show the usual 1's which result when dividing a number into itself. For example, when 7 cancels 7, the quotient 1 could be shown beside each of the 7's. However, since 1 as a factor has the same effect whether it is written in or simply understood, the 1's need not be written.

2. What is the reciprocal of $\frac{3}{8}$?

$$1 \div \frac{3}{8} = \frac{8}{8} \div \frac{3}{8}$$

$$= 8 \div 3, \text{ or } \frac{8}{3}$$

Check:

$$\frac{\cancel{3}}{\cancel{8}} \times \frac{\cancel{8}}{\cancel{3}} = 1.$$

3. What is the reciprocal of $\frac{5}{2}$?

SOLUTION: $1 \div \frac{5}{2} = \frac{2}{2} \div \frac{5}{2}$

$$= 2 \div 5$$

$$= \frac{2}{5}$$

Check: $\frac{\cancel{5}}{\cancel{2}} \times \frac{\cancel{2}}{\cancel{5}} = 1$

4. What is the reciprocal of $3\frac{1}{8}$?

SOLUTION: $1 \div 3\frac{1}{8} = \frac{8}{8} \div \frac{25}{8}$

$$= 8 \div 25$$

$$= \frac{8}{25}$$

Check: $\frac{\cancel{25}}{\cancel{8}} \times \frac{\cancel{8}}{\cancel{25}} = 1$

The foregoing examples lead to the rule for finding the reciprocal of any number: The reciprocal of a number is the fraction formed when 1 is divided by the number. (If the final result is a whole number, it can be considered as a fraction whose denominator is 1.) A short-cut rule which is purely mechanical and does not involve reasoning may be stated as follows: To find the reciprocal of a number, express the number as a fraction and then invert the fraction.

When the numerator of a fraction is 1, the reciprocal is a whole number. The smaller the fraction, the greater is the reciprocal. For example, the reciprocal of $\frac{1}{1,000}$ is 1,000. Also, the reciprocal of any whole number is a proper fraction. Thus the reciprocal of 50 is $\frac{1}{50}$.

Practice problems. Write the reciprocal of each of the following numbers:

1. 4 2. $\frac{1}{3}$ 3. $2\frac{1}{2}$ 4. 17 5. $\frac{3}{2}$ 6. $\frac{5}{1}$

Answers:

1. $\frac{1}{4}$ 2. 3 3. $\frac{2}{5}$ 4. $\frac{1}{17}$ 5. $\frac{2}{3}$ 6. $\frac{1}{5}$

The reciprocal method of division makes use of the close association of multiplication and division. In any division problem, we must find the answer to the following question: What number multiplied by the divisor yields the dividend? For example, if the problem is to divide 24 by 6, we must find the factor which, when multiplied by 6, yields 24. Experience tells us that the number we seek is 1/6 of 24. Thus, we may rewrite the problem as follows:

$$24 \div 6 = \frac{1}{6} \times 24$$

$$= \frac{1 \times \overset{4}{\cancel{24}}}{\cancel{6} \times 1}$$

$$= 4$$

Check: $6 \times 4 = 24$

In the example $1\frac{1}{2} \div 3$, we could write $3 \times ? = 1\frac{1}{2}$. The number we seek must be one-third of $1\frac{1}{2}$. Thus we can do the division by taking one-third of $1\frac{1}{2}$; that is, we multiply $1\frac{1}{2}$ by the reciprocal of 3.

$$1\frac{1}{2} \div 3 = 1\frac{1}{2} \times \frac{1}{3}$$

$$= \frac{\cancel{3}}{2} \times \frac{1}{\cancel{3}}$$

$$= \frac{1}{2}$$

Check: $3 \times \frac{1}{2} = \frac{3}{2} = 1\frac{1}{2}$

The rule for division by the reciprocal method is: Multiply the dividend by the reciprocal of the divisor. This is sometimes stated in short form as follows: Invert the divisor and multiply.

The following examples of cases that arise in division with fractions will be solved by both the reciprocal method and the common denominator method. The common denominator method more clearly shows the division process and is easier for the beginner to grasp. The reciprocal method is more obscure as to the reason for its use but has the advantage of speed and the possibility of cancellation of like factors, which simplifies the computation. It is the suggested method once the principles become familiar.

EXAMPLE: $\frac{2}{5} \div 4 = ?$

Common Denominator Method

$$\frac{2}{5} \div 4 = \frac{2}{5} \div \frac{20}{5}$$

$$= 2 \div 20$$

$$= \frac{2}{20} = \frac{1}{10}$$

Reciprocal Method

$$\frac{2}{5} \div 4 = \frac{2}{5} \times \frac{1}{4}$$

$$= \frac{\cancel{2} \times 1}{5 \times \underset{2}{\cancel{4}}}$$

$$= \frac{1}{10}$$

EXAMPLE: $2\frac{2}{3} \div 3 = ?$

Common Denominator Method

$$2\frac{2}{3} \div 3 = \frac{8}{3} \div \frac{9}{3}$$

$$= 8 \div 9$$

$$= \frac{8}{9}$$

Reciprocal Method

$$2\frac{2}{3} \div 3 = \frac{8}{3} \times \frac{1}{3}$$

$$= \frac{8 \times 1}{3 \times 3}$$

$$= \frac{8}{9}$$

EXAMPLE: $9 \div \frac{2}{7} = ?$

Common Denominator Method

$$9 \div \frac{2}{7} = \frac{63}{7} \div \frac{2}{7}$$

$$= 63 \div 2$$

$$= \frac{63}{2} = 31\frac{1}{2}$$

Reciprocal Method

$$9 \div \frac{2}{7} = 9 \times \frac{7}{2}$$

$$= \frac{9 \times 7}{1 \times 2}$$

$$= \frac{63}{2} = 31\frac{1}{2}$$

EXAMPLE: $10 \div 5\frac{3}{4} = ?$

Common Denominator Method

$$10 \div 5\frac{3}{4} = \frac{40}{4} \div \frac{23}{4}$$
$$= 40 \div 23$$
$$= \frac{40}{23} = 1\frac{17}{23}$$

Reciprocal Method

$$10 \div 5\frac{3}{4} = 10 \times \frac{4}{23}$$
$$= \frac{10 \times 4}{1 \times 23}$$
$$= \frac{40}{23} = 1\frac{17}{23}$$

EXAMPLE: $\frac{2}{3} \div \frac{1}{4} = ?$

Common Denominator Method

$$\frac{2}{3} \div \frac{1}{4} = \frac{8}{12} \div \frac{3}{12}$$
$$= 8 \div 3$$
$$= \frac{8}{3} = 2\frac{2}{3}$$

Reciprocal Method

$$\frac{2}{3} \div \frac{1}{4} = \frac{2}{3} \times \frac{4}{1}$$
$$= \frac{8}{3} = 2\frac{2}{3}$$

EXAMPLE: $\frac{9}{16} \div \frac{3}{10} = ?$

Common Denominator Method

$$\frac{9}{16} \div \frac{3}{10} = \frac{45}{80} \div \frac{24}{80}$$
$$= 45 \div 24$$
$$= \frac{45}{24} = \frac{15}{8}$$
$$= 1\frac{7}{8}$$

Reciprocal Method

$$\frac{9}{16} \div \frac{3}{10} = \frac{9}{16} \times \frac{10}{3}$$
$$= \frac{\overset{3}{\cancel{9}}}{\underset{8}{\cancel{16}}} \times \frac{\overset{5}{\cancel{10}}}{\cancel{3}}$$
$$= \frac{15}{8} = 1\frac{7}{8}$$

Practice problems. Perform the following division by the reciprocal method:

1. $\frac{3}{8} \div \frac{2}{3}$ 2. $2\frac{1}{3} \div 1\frac{1}{2}$ 3. $\frac{5}{8} \div \frac{5}{16}$ 4. $\frac{1}{3} \div \frac{4}{6}$

Answers:

1. $\frac{9}{16}$ 2. $1\frac{5}{9}$ 3. 2 4. $\frac{1}{2}$

COMPLEX FRACTIONS

When the numerator or denominator, or both, in a fraction are themselves composed of fractions, the resulting expression is called a complex fraction. The following expression is a complex fraction:

$$\frac{3/5}{3/4}$$

This should be read "three-fifths over three-fourths" or "three-fifths divided by three-fourths." Any complex fraction may be simplified by writing it as a division problem, as follows:

$$\frac{3/5}{3/4} = \frac{3}{5} \div \frac{3}{4}$$
$$= \frac{\cancel{3}}{5} \cdot \frac{4}{\cancel{3}}$$
$$= 4/5$$

Similarly,

$$\frac{3\frac{1}{3}}{2\frac{1}{2}} = \frac{10}{3} \div \frac{5}{2} = \frac{\overset{2}{\cancel{10}}}{3} \times \frac{2}{\cancel{5}} = \frac{4}{3} = 1\frac{1}{3}$$

Complex fractions may also contain an indicated operation in the numerator or denominator or both. Thus,

$$\frac{\frac{1}{2} + \frac{1}{3}}{\frac{9}{5} + \frac{1}{5}}$$

is a complex fraction. To simplify such a fraction we simplify the numerator and denominator and proceed as follows:

$$\frac{\frac{1}{2} + \frac{1}{3}}{\frac{9}{5} + \frac{1}{5}} = \frac{\frac{3}{6} + \frac{2}{6}}{\frac{10}{5}} = \frac{\frac{5}{6}}{2}$$
$$= \frac{5}{6} \div \frac{2}{1}$$
$$= \frac{5}{6} \times \frac{1}{2}$$
$$= \frac{5}{12}$$

Mixed numbers appearing in complex fractions usually show the plus sign.

Thus,

$$4\frac{2}{5} \div 7\frac{1}{3}$$

might be written

$$\frac{4 + \frac{2}{5}}{7 + \frac{1}{3}}$$

Practice problems. Simplify the following complex fractions:

1. $\dfrac{1}{\frac{3}{8}}$ 2. $\dfrac{2\frac{1}{2}}{3}$ 3. $\dfrac{3\frac{2}{3}}{2\frac{2}{5}}$ 4. $\dfrac{\frac{1}{4} + \frac{1}{3}}{\frac{1}{16} - \frac{1}{32}}$

Answers:

1. $2\frac{2}{3}$ 2. $\frac{5}{6}$ 3. $1\frac{19}{36}$ 4. $18\frac{2}{3}$

Complex fractions may arise in electronics when it is necessary to find the total resistance of several resistances in parallel as shown in figure 4-8. The rule is: The total resistance of a parallel circuit is 1 divided by the sum of the reciprocals of the separate resistances. Written as a formula, this produces the following expression:

$$R_t = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

EXAMPLE: Find the total resistance of the parallel circuit in figure 4-8 (A). Substituting the values 3, 4, and 6 for the letters R_1, R_2, and R_3, we have the following:

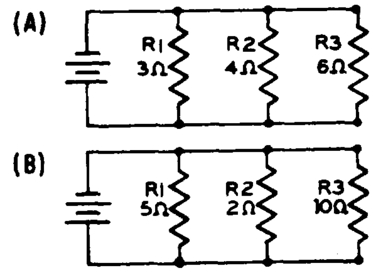


Figure 4-8.—Application of complex fractions in calculating electrical resistance.

$$R_t = \frac{1}{\frac{1}{3} + \frac{1}{4} + \frac{1}{6}}$$

The LCD of the fractions $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{6}$ is 12.

Thus,

$$R_t = \frac{1}{\frac{4}{12} + \frac{3}{12} + \frac{2}{12}}$$

$$= \frac{1}{\frac{9}{12}}$$

$$= \frac{12}{9} = \frac{4}{3}$$

$$= 1\frac{1}{3} \text{ ohms (measure of resistance).}$$

Practice problem: Find the total resistance of the parallel circuit in figure 4-8 (B).

Answer: $1\frac{1}{4}$ ohms.